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OVER-ALL ASYMPTOTIC STABILITY OF SELF-TUNING SYSTEMS WITH A REFERENCE MODEL

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Sufficient conditions of over-all asymptotic stability are established with respect to coordinate and parameter mismatch for additive systems with a reference model. Application of derived conditions is shown on the example of a second order system.

One of the fundamental properties that a self-tuning system must possess is that of asymptotic stability. It is important that the self-tuning circuit ensures not only the system asymptotic stability at small a priori unknown variation of the object parameters but, also, at any initial deviations, i.e. that it guarantees the over-all stability of the system [1]. It should be borne in mind that the asymptotic stability of additive systems with respect to tuned parameters depends on the mode of control actions. With some kinds of such actions an asymptotic stability with respect to parameter mismatch may not be feasible.

Certain conditions that ensure a uniform asymptotic stability of an additive system with a model were obtained in [2], but conditions of over-all uniform asymptotic stability were not obtained there.

Below we determine the conditions that must be imposed on the reference model, the self-tuning circuit, and on the control action which would ensure the over-all asymptotic stability of a self-tuning system with respect to coordinate and parameter mismatch.

1. Let the equations of the system and of its reference model be of the form

$$x^{\cdot}(t) = Ax(t) + [\Delta A + \delta A(t, x, y)]\, x(t) + f(t),\ x(t_0) = x_0 \tag{1.1}$$

$$y^{\cdot}(t) = Ay(t) + f(t),\ y(t_0) = y_0 \tag{1.2}$$

where $x(t) \in R^n$ and $y(t) \in R^n$ are phase coordinate vectors of the system and reference model, respectively; A is a real constant stable $n \times n$ matrix; ΔA is a real constant $n \times n$ matrix whose coefficients dependent on the control objectives are a priori unknown, and $\delta A(t, x, y)$ is an $n \times n$ matrix of parameters that are affected by the self-tuning circuit. Matrices A, ΔA, and $\delta A(t, x, y)$ are of the form

$$A = \begin{Vmatrix} 0 & 1 & 0 \ldots 0 & 0 \\ 0 & 0 & 1 \ldots 0 & 0 \\ \cdot & \cdot & \cdot \cdot \cdot \cdot & \cdot \\ 0 & 0 & 0 \ldots 0 & 1 \\ a_1 & a_2 & a_3 \ldots a_{n-1} & a_n \end{Vmatrix}$$

$$\Delta A = \begin{Vmatrix} 0 \ldots 0 \\ \cdot \cdot \cdot \cdot \cdot \\ 0 \ldots 0 \\ \Delta a_1 \ldots \Delta a_n \end{Vmatrix}, \quad \delta A (t, x, y) = \begin{Vmatrix} 0 \ldots \ldots \ldots 0 \\ \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \\ 0 \ldots \ldots \ldots 0 \\ \delta a_1 (t, x, y) \ldots \delta a_n (t, x, y) \end{Vmatrix} \tag{1.3}$$

and the vector of control actions is of the form

$$f' (t) = (0, \ldots, 0, f_n (t))$$

where a prime denotes the transposition of a vector or matrix.

The matrix form of the n-th order equations of systems (1.1) and (1.2) does not restrict the generality of exposition (any linear system can be reduced to a single equation of a higher order).

Subtracting (1.2) from (1.1), for the parameter mismatch vector $\varepsilon (t) = x (t) - y (t)$ we obtain the equation

$$\varepsilon^{\cdot} (t) = A \varepsilon (t) + [\Delta A + \delta A (t, x, y)] x (t), \quad \varepsilon (t_0) = \varepsilon_0$$

or in the form more convenient for further analysis

$$\varepsilon^{\cdot} (t) = A \varepsilon (t) + X (t) \alpha (t, x, y), \quad \varepsilon (t_0) = \varepsilon_0 \tag{1.4}$$

where $X (t)$ is an $n \times n$ matrix of the form of ΔA in (1.3) in which the last row coincides with vector $x'(t)$, and $\alpha (t, x, y)$ is the parameter mismatch vector of the form

$$\alpha' (t, x, y) = (\Delta a_1 + \delta a_1 (t, x, y), \ldots, \Delta a_n + \delta a_n (t, x, y))$$

Let us consider the self-tuning circuit defined by a following equation

$$\alpha^{\cdot} (t) = - X' (t) \Gamma (t, x, y) \varepsilon (t), \ \alpha' (t_0) = \alpha_0' = (\Delta a_1, \ldots, \Delta a_n) \tag{1.5}$$

where $\Gamma(t, x, y)$ is an $n \times n$ positive definite symmetric matrix whose properties completely determine the self-tuning circuit.

The necessity to select the form (1.5) for the self-tuning circuit was considered in detail in [1, 2]. Here we shall only point out the theoretical importance of the independence of the right-hand side of (1.5) from the unknown and nonmeasurable parameter mismatch vector $\alpha (t)$. Absence of the latter from the right-hand side of (1.5) results in considerable difficulties in the analysis of stability of system (1.4), (1.5).

The problem is stated as follows: determine the sufficient conditions of the overall asymptotic stability of the ststem of Eqs. (1.4) and (1.5).

To solve that problem we use a single general theorem stated below.

2. Let us consider the following Cauchy's problem:

$$z^{\cdot}(t) = F(z, t), \quad F(0, t) \equiv 0, \quad z(t_0) = z_0 \tag{2.1}$$

$$F : R^m \times [0, \infty) \to R^m$$

where F is a certain nonlinear operator which satisfies the conditions under which the theorems about local existence and uniqueness of solution of problem (2.1) are valid.

We denote by $\langle z, y\rangle$ the scalar product in R^m, by $\| z \|$ the Euclidean norm of vector $z \in R^m$, and by $d(z, G)$ the distance between vector $z \in R^m$ and some set $G \subset R^m$.

Definition. The trivial solution of system (2.1) is considered over-all asymptotically stable uniformly over initial data, if it is uniformly stable at the initial instant t_0 and, if for any sphere $S_k = \{z \in R^m : \| z \| \leqslant K\}$ and any number $\gamma > 0$ there exists a number $T(\gamma, K) > 0$ such that for all $t \geqslant t_0 + T(\gamma, K)$ and any $t_0 \geqslant 0$ and any $z_0 \in S_k$, $\| z(t) \| \leqslant \gamma$ [3].

Theorem 1. Let us assume that there exist two continuously differentiable functions $V(z, t)$ and $W(z, t)$, $V, W : R^m \times [0, \infty) \to R^1$ which in any arbitrary sphere S_k have the following properties:

A) $\omega_1(\| z \|) \leqslant V(z, t) \leqslant \omega_2(\| z \|)$
where $\omega_1(u)$ and $\omega_2(u)$ are continuous nondecreasing functions such that $\omega_1(0) = \omega_2(0) = 0$, $\omega_1(u) > 0$, $\omega_2(u) > 0$, $u \neq 0$ and $\omega_1(u) \to \infty$ when $u \to \infty$;

B) in solutions of system (2.1)

$$V^{\cdot}(t) = \frac{dV}{\partial t} + < \operatorname{grad} V, \quad F > \leqslant \omega_3(z) \leqslant 0$$

where $\omega_3(z)$ is a continuous nonpositive function in R^m and $O(\omega_3 = 0)$, denotes a set in R^m for which $\omega_3(z) = 0$;

C) for $0 \leqslant t < \infty$ and all $z \in S_k$ function $W(z, t)$ is bounded and $| W(z, t) | \leqslant L(K)$;

D) for some number $B > 2L(K)$, any $t_* \geqslant t_0 > 0$ and any number μ, $0 < \mu < K$ there exist numbers $T(B)$, $0 < T(B) < \infty$ and $\rho_0(\mu, B, T(B))$ such that

$$W^{\cdot}(t) = \partial W / \partial t + < \operatorname{grad} W, \quad F > = \xi(z, t)$$

satisfies uniformly with respect to $z \in E(\mu, \rho)$ the inequality

$$\left| \int_{t_*}^{t_*+T(B)} \xi(z(s), s)\, ds \right| \geqslant B \tag{2.2}$$

where $0 < \rho < \rho_0(\mu, B, T, (B))$ and $E(\mu, \rho)$ is a set of the form

$$E(\mu, \rho) = \{\mu \leqslant \| z \| \leqslant K, \ d(z, O(\omega_3 = 0)) \leqslant \rho\}$$

The trivial solution of system (2.1) is then over-all asymptotically stable with respect to initial data.

Theorem 1 represents a certain enhancement of the known stability criterion [4]. Stipulation a) may be otherwise specified as a requirement for function $V(z, t)$ to be a positive definite and allow an infinitely small upper and infinitely large lower limits. Requirements A), B), and C) coincide with those of Matrosov's theorem, while D) is less stringent than the corresponding conditions stipulated in [4].

The proof of Theorem 1 differs from that of Matrosov's [4] only by the lemma on "discarding", a definition proposed in [5]. Hence we shall only present the proof of that lemma.

Lemma on "discarding". The perturbed motion $z(t)$ of system (2.1) cannot constantly remain in the set $E(\mu, \rho)$ during time $T(B)$, where $E(\mu, \rho)$, B, and $T(B)$ are determined by condition D) of Theorem 1.

Proof. Let us assume that the perturbed motion $z(t)$ remains in the set $E(\mu, \rho)$ during time $T(B)$. Then, using (2.2), we obtain that

$$|W(z(t+T(B)), t+T(B)) - W(z(t), t)| = \tag{2.3}$$
$$= \left| \int_{t_*}^{t_*+T(B)} \xi(z(s), s)\, ds \right| \geqslant B$$

which contradicts condition D) of Theorem 1.

3. We shall use Theorem 1 for determining conditions of the over-all asymptotic stability of the trivial solution of system (1.4), (1.5), which is uniform with respect to initial data. Let us consider function

$$V(\varepsilon, \alpha, t) = \varepsilon' \Gamma(t, x, y)\, \varepsilon + \alpha' \alpha \tag{3.1}$$

where matrix $\Gamma(t, x, y)$ is the same as in Eq. (1.5). The total derivative of function (3.1) with respect to time, determined by solutions of system (1.4), (1.5) is

$$V^{\cdot}(t) = \varepsilon' (A'\Gamma + \Gamma A + \Gamma^{\cdot})\, \varepsilon \tag{3.2}$$

Let us assume that the conditions

$$C_1 \| p \|^2 \leqslant p' \Gamma(t, x, y)\, p \leqslant C_2 \| p \|^2 \tag{3.3}$$
$$p' (A'\Gamma + \Gamma A + \Gamma^{\cdot})\, p \leqslant - C_3 \| p \|^2 \tag{3.4}$$

are satisfied for all $t \geqslant 0$ and any arbitrary vector $p \in R^n$. In these equations and in what follows C_i denote positive constants whose values are unimportant.

From (3.3) we obtain that

$$\| \Gamma(t, x, y) \| \leqslant C_4 \tag{3.5}$$

Note that there exist, for instance, constant matrices that satisfy conditions (3.3) and (3.4) [6].

We shall show that function (3.1) satisfies stipulations A) and B) of Theorem 1. Evidently

$$C_5 (\| \varepsilon \|^2 + \| \alpha \|^2) \leqslant V(\varepsilon, \alpha, t) \leqslant C_6 (\| \varepsilon \|^2 + \| \alpha \|^2)$$
$$(C_5 = \min (C_1. 1), \ C_6 = \max (C_2, 1)) \qquad (3.6)$$

which proves that requirement A) of Theorem 1 is satisfied, and requirement B) immediately follows from condition (3.4). The set O $(\omega_3 = 0)$ is a hyperplane of the form $\varepsilon = 0$ in space R^{2n}.

Function (3.1) satisfies conditions of the theorem of Persidskii [3] which implies that the trivial solution of system (1.4), (1.5) is uniformly stable with respect to t_0. This shows that for all $t \geqslant t_0 \geqslant 0$, $\| \varepsilon_0 \| \leqslant K$, $\| \alpha_0 \| \leqslant K$

$$\| \varepsilon (t) \|^2 + \| \alpha (t) \|^2 \leqslant \frac{1}{C_5} V (\varepsilon_0, \alpha_0, t_0) \leqslant \frac{C_6}{C_5} K^2 = D^2 (K)$$

or

$$\| \varepsilon (t) \| \leqslant D (K), \quad \| \alpha (t) \| \leqslant D (K) \qquad (3.7)$$

As the second function $W(\varepsilon, \alpha, t)$ we take function

$$W (\varepsilon, \alpha, t) = \varepsilon' X (t) \alpha \qquad (3.8)$$

We assume that the vector of control actions is bounded, i.e.

$$\| f (t) \| \leqslant F_0, \quad 0 \leqslant t_0 \leqslant t < \infty \qquad (3.9)$$

Owing to the asymptotic stability of system (1.2), vector $y(t)$ is also bounded for all $0 \leqslant t_0 \leqslant t < \infty$ and $\| y_0 \| \leqslant H$, i.e.

$$\| y (t) \| \leqslant Y \qquad (3.10)$$

Since $x(t) = y(t) + \varepsilon(t)$, hence, taking into account (3.7) and (3.10) we obtain that for $t \geqslant t_0$, $\| \varepsilon_0 \| \leqslant K$, $\| \alpha_0 \| \leqslant K$, $\| y_0 \| \leqslant H$

$$\| x.(t) \| = \| X (t) \| \leqslant \| y (t) \| + \| \varepsilon (t) \| \leqslant Y + D (K) = X_1(K) \qquad (3.11)$$

which implies that function W of the form (3.8) is bounded in any arbitrary sphere S_k, since

$$| W (\varepsilon, \alpha, t) | \leqslant \| \varepsilon (t) \| \, \| X (t) \| \, \| \alpha (t) \| \leqslant$$
$$D^2 (K) X_1 (K) = L (K) \qquad (3.12)$$

The total derivative of function (3.8) with respect to time, obtained from the solution of system (1.4), (1.5) is

$$W^{\bullet} (t) = \alpha' X' X \alpha + \varepsilon' (A' X \alpha + X^{\bullet} \alpha - X X' \Gamma \varepsilon) = \langle \alpha, y \rangle^2 +$$
$$\varepsilon' (A' X \alpha + X^{\bullet} \alpha - X X' \Gamma \varepsilon) + 2 \langle \alpha, y \rangle \langle \alpha, \varepsilon \rangle + \langle \alpha, \varepsilon \rangle^2 \qquad (3.13)$$

since

$$\alpha' X' X \alpha = \langle \alpha, x \rangle^2 = \langle \alpha, y + \varepsilon \rangle^2$$

All quantities in the right-hand side of (3.13) are calculated for the same instant of time t.

From Eq. (1.5) we obtain that for any t and t_*, $t \geqslant t_* \geqslant t_0$,

$$\alpha(t) = \alpha(t_*) - \int_{t_*}^{t} X'(s)\,\Gamma(s, x(s), y(s))\,\varepsilon(s)\,ds \tag{3.14}$$

The substitution of (3.14) into (3.13) yields for the derivative of function $W(\varepsilon, \alpha, t)$ the following formula:

$$\begin{aligned} W^{\bullet}(t) = {} & \langle \alpha(t_*), y(t)\rangle^2 + \varepsilon' [A'X\alpha + X^{\bullet}\alpha - XX'\Gamma\varepsilon] + \\ & \langle \alpha(t), \varepsilon(t)\rangle^2 + 2\langle \alpha(t), y(t)\rangle \langle \alpha(t), \varepsilon(t)\rangle + \\ & \left\langle \int_{t_*}^{t} X'(s)\,\Gamma(s, x(s), y(s))\,\varepsilon(s)\,ds,\ y(t)\right\rangle^2 - \\ & 2\left\langle \int_{t_*}^{t} X'(s)\,\Gamma(s, x(s), y(s))\,\varepsilon(s)\,ds,\ y(t)\right\rangle \langle \alpha(t_*), y(t)\rangle \end{aligned} \tag{3.15}$$

Equation (1.1) and conditions (3.7), (3.9), and (3.11) imply that

$$\begin{aligned} \| X^{\bullet}(t) \| = \| x^{\bullet}(t) \| \leqslant \| A \| \, \| x(t) \| + \| \alpha(t) \| \, \| x(t) \| + \\ \| f(t) \| \leqslant (\| A \| + D(K))\, X_1(K) + F_0 = X_2(K) \end{aligned} \tag{3.16}$$

Let us now assume that the solution $y(t) = 0$ of the reference model (1.2) is such that for any constant vector $\eta \quad \alpha(t_*), \eta \in R^n, \| \eta \| \geqslant \mu > 0$, any $t_* \geqslant t_0$, and some number $B > L(K)$ there exists a $T(B)$ such that

$$\int_{t_*}^{t_*+T(B)} \langle \eta, y(s)\rangle^2\, ds \geqslant 3B \tag{3.17}$$

For (3.17) to be valid the vector components must be linearly independent along some arbitrary segment $[t_*, t_* + T(B)]$. A further requirement is that the scalar product of vector $y(t)$ by an arbitrary constant vector $\eta, \| \eta \| \geqslant \mu > 0$ must not tend to vanish too rapidly when $t \to \infty$. Thus (3.17) is an implicid condition imposed on the form of control action $f(t)$.

It follows from (3.15) that the inequality

$$\begin{aligned} W^{\bullet}(t) \geqslant {} & \langle \eta, y(t)\rangle^2 - \rho\,(\| A \|\, X_1(K)\, D(K) + X_2(K)\, D(K) + \\ & X_1^2(K)\, \| \Gamma \|\, \rho) - 2\rho D^2(K)\, Y - \rho^2 D^2(K) - \rho^2 X_1^2(K) \times \\ & \| \Gamma \|^2\, Y^4 T^2(B) - 2\rho X_1(K)\, \| \Gamma \|\, Y^2 D(K)\, T(B) = \\ & \langle \eta, y(t)\rangle^2 - \rho N(K, Y, T(B)) \end{aligned} \tag{3.18}$$

holds for all t, $0 \leqslant t_0 \leqslant t_* \leqslant t \leqslant t_* + T(B)$ such that $\| \varepsilon(t) \| \leqslant \rho$ and $\mu \leqslant \| \alpha(t) \| \leqslant K$, uniformly on $\varepsilon(t)$ and $\alpha(t)$.

By virtue of (3.17) and (3.18) we have

$$\int_{t_*}^{t_*+T(B)} W^{\cdot}(s)\,ds \geqslant \int_{t_*}^{t_*+T(B)} \langle \eta, y(s)\rangle^2\,ds - \rho T(B)\,N(K, Y, T(B)) > 2B > 2L(K) \tag{3.19}$$

which is valid for any constant vector $\eta \in R^n$, $B > L(K)$, and $\rho \leqslant \rho_0$ such that $\rho_0 T(B)\,N(K, Y, T(B)) = {}^1/_2\,B$.

It follows from (3.18) and (3.19) that function $W(\varepsilon, \alpha, t)$ satisfies condition D) of Theorem 1.

Using Theorem 1 and the known theorem on stability under conditions of constantly acting perturbations [7], we obtain the following theorem.

Theorem 2. We assume that A is a constant matrix, the control function $f(t)$ is bounded, and that $\| y_0 \| \leqslant H$. If solution $y(t)$ of the reference model [equation] satisfies condition (3.17) and matrix $\Gamma(t, x, y)$ satisfies conditions (3.3) and (3.4), the trivial solution of system (1.4), (1.5) is then over-all stable with respect to initial data. Furthermore, such trivial solution is stable under constant action of perturbations.

Systems in which only some of parameters a_i $(i = 1, \ldots, n)$ vary are encountered in practical applications. The self-tuning circuit of such systems must be designed to control only the variable parameters. Conditions of the over-all asymptotic stability of such systems are obtained as the corollary of Theorem 1.

When the reference model is defined by Eq. (1.2) and only parameters at places defined by $i_1, \ldots, i_k$ in the system received a priori unknown increments, the system can be defined by the equation

$$x^{\cdot}(t) = Ax(t) + [\Delta A_* + \delta A_*(t, x, y)]\,x_*(t) + f(t), \quad x(t_0) = x_0 \tag{3.20}$$

where the $n \times n$ matrices ΔA_* and $\delta A_*(t, x, y)$ are of the form (1.3) and the only nonzero elements in the last row of these matrices appear at places denoted by $i_1, \ldots, i_k$, with $k \leqslant n$. The $n \times 1$ vector whose components at places $i_1, \ldots, i_k$ coincide with those of vector $x(t)$ are denoted by $x_*(t)$. All other components are zero.

The coordinate mismatch vector $\varepsilon(t) = x(t) - y(t)$ satisfies the equation

$$\varepsilon^{\cdot}(t) = A\varepsilon(t) + X_*(t)\,\alpha(t, x, y), \quad \varepsilon(t_0) = \varepsilon_0 \tag{3.21}$$

where $\alpha_*'(t, x, y)$ is the $1 \times n$ vector of parameter mismatch and coincides with the last row of matrix $\Delta A_* + \delta A_*(t, x, y)$ and $X_*(t)$ is an $n \times n$ matrix whose all rows are zero, except the last one which is the same as vector $x_*'(t)$.

The algorithm of self-tuning is defined by

$$\alpha_*^{\cdot}(t) = -X_*'(t)\,\Gamma(t, x, y)\,\varepsilon(t) \tag{3.22}$$
$$\alpha_*'(t_0) = \alpha_{0*}' = \{\Delta A_*\}_n$$

All conditions of Theorem 1, except (3.17), apply to system (3.21), (3.22) in their previous form. Condition (3.17) is defined in this case as follows. For any $t_* \geqslant t_0 \geqslant 0$ and any vector η_*, $\|\eta_*\| \geqslant \mu > 0$ whose components, except the $i_1, \ldots, i_k$-th, are zero, the reference model vector $y(t)$ must be such that

$$\int_{t_*}^{t_*+T(B)} \langle \eta_*, y(s)\rangle^2 ds = \int_{t_*}^{t_*+T(B)} \langle \eta_*, y_*(s)\rangle^2 ds \geqslant 3B \qquad (3.23)$$

where $y_*(s)$ is a vector whose $i_1, \ldots, i_k$-th components are the same as the components of vector $y(s)$, and all remaining are zero. Thus in the considered case the less stringent condition (3.23) is substituted for (3.17).

The conditions of asymptotic stability of system (3.21), (3.22) can be stated in the form of the following theorem.

Theorem 3. Let all conditions of Theorem 1, except (3.17) replaced by condition (3.23), be satisfied. The trivial solution of system (3.21), (3.22) is then over-all asymptotically stable uniformly with respect to initial data.

4. Let us show on an example the stringency of conditions (3.17) and of (3.23).

Let the reference model and the system be defined, respectively, by equations of the form (4.1) and (4.2)

$$y^{\cdot\cdot}(t) + 2\delta y^{\cdot}(t) + ay(t) = f(t), \quad y(t_0) = y_0, \ y^{\cdot}(t_0) = y_1 \qquad (4.1)$$

$$x^{\cdot\cdot}(t) + 2[\delta + \alpha_1(t, x, y)]\, x^{\cdot}(t) + [a + \alpha_0(t, x, y)]\, x(t) = f(t) \qquad (4.2)$$

Equation (4.1) is asymptotically stable when $\delta > 0$ and $a > 0$.

If $f(t) \equiv 1$, the solution of Eq. (4.1) is of the form $y(t) = a^{-1} + \beta_1 y_1(t) + \beta_2 y_2(t)$, $y^{\cdot}(t) = \beta_1 y_1^{\cdot}(t) + \beta_2 y_2^{\cdot}(t)$, where $y_1(t)$ and $y_2(t)$ are two linearly independent solutions of that equation, and β_1 and β_2 are constants determined by initial conditions. Obviously

$$|y_1(t)| + |y_1^{\cdot}(t)| \leqslant C_7 \exp(-C_9 t), \quad C_7, C_9 > 0$$
$$|y_2(t)| + |y_2^{\cdot}(t)| \leqslant C_8 \exp(-C_{10} t), \quad C_8, C_{10} > 0$$

Let us consider the scalar product $\langle \eta, y(t)\rangle = \eta_1 y(t) + \eta_2 y^{\cdot}(t)$ of the arbitrary vector $\eta' = (\eta_1, \eta_2)$, $\eta_1^2 + \eta_2^2 \geqslant \mu^2$. If $\eta_1 = 0$ and $\eta_2 = \mu$, then

$$\int_{t_*}^{t_*+T(B)} \eta_2^2 (y^{\cdot}(s))^2 ds \leqslant \mu^2 \int_0^\infty (\beta_1 y_1^{\cdot}(s) + \beta_2 y_2^{\cdot}(s))^2 \leqslant$$
$$2\mu^2 \int_0^\infty (\beta_1^2 (y_1^{\cdot}(s))^2 + \beta_2^2 (y_2^{\cdot}(s)))^2 ds \leqslant 2\mu^2 \left(\frac{\beta_1^2 C_7^2}{2C_9} + \frac{\beta_2^2 C_8^2}{2C_{10}}\right)$$

Thus in this case condition (3.17) is not satisfied for fairly large numbers B, and there may be no over-all asymptotic stability with respect to all control parameters of system (4.2) with the reference model (4.1).

But, if the system is of the form

$$x^{\cdot\cdot}(t) + 2\delta x^{\cdot}(t) + [a + \alpha_0(t, x, y)]x(t) = f(t) \tag{4.3}$$

it is possible to use Theorem 3. Condition (3.23) is then satisfied when $f(t) \equiv 1$, since for any $t_*, \eta_*, \|\eta_*\| \geqslant \mu > 0$, $B > 0$, and $T(B) \geqslant 3Ba/\mu^2$

$$\int_{t_*}^{t_*+T(B)} \langle \eta_*, y_*^{\cdot}(s)\rangle^2 ds = \int_{t_*}^{t_*+T(B)} \eta_1^2 y^2(s)\, ds \geqslant \mu^2 \frac{1}{a} T(B) \geqslant 3B$$

Hence in this case we have asymptotic stability with respect to parameter α_0.

Let

$$f(t) = A_0 + \sum_{k=1}^{k} (A_k \sin\omega_k t + B_k \cos\omega_k t), \quad \omega_k \neq 0 \tag{4.4}$$

The solution of Eq. (4.1) is now of the form

$$y(t) = \frac{A_0}{2} + \sum_{k=1}^{n} (E_k \sin\omega_k t + D_k \cos\omega_k t) + \beta_3 y_1(t) + \beta_4 y_2(t)$$

$$E_k = \frac{A_k(a-\omega_k^2) + 2B_k\delta\omega_k}{(a-\omega_k^2)^2 + 4\delta^2\omega_k^2}, \quad D_k = \frac{B_k(a-\omega_k^2) - 2A_k\delta\omega_k}{(a-\omega_k^2)^2 + 4\delta^2\omega_k^2}$$

and (3.17) is defined by

$$\int_{t_*}^{t_*+T(B)} (\eta_1 y(s) + \eta_2 y^{\cdot}(s))^2 ds = \tag{4.5}$$

$$\frac{T(B)}{2}(\eta_1^2 + \eta_2^2)\sum_{k=1}^{n}(E_k^2 + D_k^2) + g(t_*, T(B), \eta_1, \eta_2, \beta_3, \beta_4)$$

where $g(t_*, T(B), \eta_1, \eta_2, \beta_3, \beta_4)$ is a bounded function for all $t_* \geqslant 0$, $T(B) > 0$ and for all bounded η_1, η_2, β_3, and β_4. It follows from (4.5) that solution $y(t)$ of Eq. (4.1) with action (4.4) such that $A_1^2 + B_1^2 + \ldots + A_n^2 + B_n^2 > 0$ satisfies condition (3.17).

In conformity with Theorem 1 we have then an over-all asymptotic stability with respect to all control parameters and mismatch coordinates.

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